Back around 1970, when I was a junior in college, I took an introductory course in point-set topology using this book, which was originally published in 1963 by McGraw Hill. The course was such a success, and the book so popular, that the class requested the professor to continue on next semester to finish the book up. The professor arranged for a “topics” course to run, and we finished off most of the rest of the text. I distinctly remember thinking that this book was perhaps the best mathematics textbook ever written.

Of course, that was the opinion of a 20-year old, who (like most people that age) thought he knew a lot more than he actually did. With the benefit of about 45 years of additional experience, I now realize that this book isn’t quite as perfect as I thought it was; it has some definite flaws that I’ll discuss below. But it was, and still is, a very good book, and it holds up remarkably well; so well, in fact, that I was delighted to see that it is still in print (with a different publisher), and have selected it as the text for a topology course that I am currently scheduled to teach next fall. (I should point out, though, that some weeks after I made that adoption decision, I made the acquaintance of Croom's excellent book Principles of Topology; had I known of this book earlier, I likely would have selected it instead.)

The book under review is divided into three parts (entitled, respectively, “Topology”, “Operators” and “Algebras of Operators”), but I think of it is as being divided into two main areas, both mentioned in the title: the topology part (part I of the book) and the “modern analysis” part (parts II and III of the text, which together comprise a wonderful introduction to functional analysis at the undergraduate level). There are also three appendices discussing special topics.

Part I of the book covers basic point-set topology. It starts with a chapter on sets (including cardinal arithmetic and some brief discussion of the Axiom of Choice and Zorn’s lemma), and then has a chapter on metric spaces, covering the basic definitions, examples and theorems. With the properties of open sets established in this chapter, the author then proceeds to topological spaces, the natural generalization of metric spaces. After this are chapters on compactness, separation and connectedness (in that order), covering the usual material on these topics, and then there is a chapter on approximation, which is essentially devoted to a statement and proof of Stone’s generalization of the Weirstrass approximation theorem.

The “modern analysis” portion of the book (parts II and III) is essentially an introduction to functional analysis, and to this day I know of no better place for an undergraduate to learn the basics of this subject for the first time. Part II consists of four chapters: first, a chapter reviewing basic algebraic structures (groups, rings, vector spaces, algebras), then a chapter each on Banach spaces and Hilbert spaces, and finally, in a return to algebra, a chapter on finite-dimensional spectral theory (of normal operators on an inner product space). Because of the assumption of finite-dimensionality, this final chapter is pure linear algebra, with no topology or analysis involved, and is intended to motivate the contents of part III of the text. Here, Simmons discusses Banach algebras, particularly commutative ones; the discussion eventually leads to commutative C*- algebras and a version of the spectral theorem for normal operators on a Hilbert space.

Measure theory is never mentioned in parts II and III of the text, so some of the more interesting Banach spaces (notably the L^p spaces) do not appear, but even without these examples, the student will learn a good chunk of the basic theory of functional analysis, including the “big” theorems on the subject (Uniform Boundedness, Closed Graph, Hahn-Banach, etc.), and will be nicely situated for perusal of more sophisticated texts on the subject, such as Conway's A Course in Abstract Analysis.

There are three appendices following Part III of the book under review; they discuss, in order, fixed point theorems, the Hahn-Mazurkiewicz theorem, and the Stone representation theorem for Boolean algebras. The extent to which things are actually proved in these appendices varies. In the first, some results (the Brouwer and Schauder fixed point theorems) are stated without proof; however, the Banach contraction mapping principle is proved, as is, as an application of it, the Picard existence-uniqueness theorem for differential equations. This is a very nice discussion, ...
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